

Dear incoming Calculus/Statistics students 2018-2019,

Welcome to our class. I **don't require** summer work of students, but I want to share with you a precalculus review packet that is good practice for incoming Calculus students. I am making it available to you as a separate pdf, with the suggestion that you try selected problems, so as to enter the fall feeling particularly secure.

Please **wait till the beginning of August**, if possible, to work on it. It's only June, so you haven't yet forgotten all the math you learned last year ;), after all. I have noticed that sometimes, students rush to do 'summer work' immediately at the end of the school year, but lose some of their prowess over the summer. So a mid-summer workout will serve you well.

Please work on the following subset of questions. You are welcome to do more, but if I wrote 'skip' in the list below, that indicates that the skill being practiced will certainly NOT be used next fall.

#1-4 Algebraic fractions: #3 is particularly similar to work you'll see in Calculus +.

SKIP 5-7

#8-10, 14-16, 18 exponent and log review

#27, 29 Solving for a variable

SKIP 30 -32.

#33, 36 Factoring

#43-50: Trig values: In calculus, we think entirely in radians, not in degrees. Familiarity with the unit circle is very important for calc.

#58: Algebraic manipulation of fractions

SKIP 59-68. They are wonderful problems, but we simply don't cover those types of problems in calculus.

#69-72 Lines We love lines in Calculus.

SKIP: 74-76. If we need circles in Calc, we'll review them quickly as needed.

#79: a hard domain and range problem. If you haven't had a problem like this, no worries -- we will review these ideas as needed.

#81-83. Composition of functions with that special  $(f(x+h)-f(x))/h$  fraction is a big calculus idea.

SKIP 84-87. We'll discuss graphing as needed.

#91, 94 inverses (for 94, did you talk about inverses as reflections over the line  $y=x$ ? If not, don't worry about this problem. I'll be re-teaching this idea.)

#97: A fun workout in connecting algebraic to geometric reasoning.

#102-104: Good examples of composition of functions.

The answers are on page 7-9.

Submitting your work is not required. The attached packet is just targeted practice, should you feel that would be helpful to you.

I hope you'll have a wonderful summer. Eat lots of ice cream and unload the dishwasher occasionally. See you in September! And, congratulations on your successes during junior year.

Best,  
Mrs. Braun  
braunr@mjbha.org

# Part I

## AP Calculus AB Summer Review Packet

Simplify

$$1. \frac{x^3 - 9x}{x^2 - 7x + 12}$$

$$2. \frac{x^2 - 2x - 8}{x^3 + x^2 - 2x}$$

$$3. \frac{\frac{1}{x} - \frac{1}{5}}{\frac{1}{x^2} - \frac{1}{25}}$$

$$4. \frac{9 - x^{-2}}{3 - x^{-1}}$$

Rationalize the denominator

$$5. \frac{2}{\sqrt{3} + \sqrt{2}}$$

$$6. \frac{4}{1 - \sqrt{5}}$$

$$7. \frac{1 - \sqrt{5}}{1 + \sqrt{3}}$$

Write each of the following expressions in the form of  $ca^pb^q$  where  $c$ ,  $p$ , and  $q$  are numbers

$$8. \frac{(2a^2)^3}{b}$$

$$10. \frac{a(2/b)}{3/a}$$

$$12. \frac{a^{-1}}{(b^{-1})\sqrt{a}}$$

$$9. \sqrt{9ab^3}$$

$$11. \frac{ab - a}{b^2 - b}$$

$$13. \left(\frac{a^{2/3}}{b^{1/2}}\right)^2 \left(\frac{b^{3/2}}{a^{1/2}}\right)$$

Solve for  $x$ . Do not use a calculator

$$14. 5^{(x+1)} = 25$$

$$16. \log_2 x = 3$$

$$15. \frac{1}{3} = 3^{2x+2}$$

$$17. \log_3 x^2 = 2 \log_3 4 - 4 \log_3 5$$

Simplify

$$18. \log_2 5 + \log_2 (x^2 - 1) - \log_2 (x - 1)$$

$$19. 2 \log_4 9 - \log_2 3$$

$$20. 3^{2 \log_3 5}$$

Simplify

$$21. \log_{10} 10^{1/2}$$

$$22. \log_{10} \frac{1}{10^x}$$

$$23. 2 \log_{10} \sqrt{x} + 3 \log_{10} x^{1/3}$$

Solve the following equations for the indicated variable

$$24. \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1, \text{ for } a$$

$$25. V = 2(ab + bc + ca), \text{ for } a$$

$$26. A = 2\pi r^2 + 2\pi rh, \text{ for positive } h$$

$$27. A = P + \pi rP, \text{ for } P$$

$$28. 2x - 2yd = y + xd, \text{ for } d$$

$$29. \frac{2x}{4\pi} + \frac{1-x}{2} = 0, \text{ for } x$$

For each equation complete the square and reduce to one of the standard forms  $y - y_1 = A(x - x_1)^2$  or  $x - x_1 = (y - y_1)^2$

$$30. y = x^2 + 4x + 3$$

$$31. 3x^2 + 3x + 2y = 0$$

$$32. 9y^2 - 6y - 9 - x = 0$$

Factor completely

$$33. x^6 - 16x^4$$

$$34. 4x^3 - 8x^2 - 25x + 50$$

$$35. 8x^3 + 27$$

$$36. x^4 - 1$$

Find all real solutions

$$37. x^6 - 16x^4 = 0$$

$$38. 4x^3 - 8x^2 - 25x + 50 = 0$$

$$39. 8x^3 + 27 = 0$$

Solve for  $x$

$$40. 3\sin^2 x = \cos^2 x; \quad 0 \leq x < 2\pi$$

$$41. \cos^2 x - \sin^2 x = \sin x; \quad -\pi < x \leq \pi$$

$$42. \tan x + \sec x = 2 \cos x; \quad -\infty < x < \infty$$

Without using a calculator, evaluate the following:

$$43. \cos 210^\circ$$

$$44. \sin \frac{5\pi}{4}$$

$$45. \tan^{-1}(-1)$$

$$46. \sin^{-1}(-1)$$

$$47. \cos \frac{9\pi}{4}$$

$$48. \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$49. \tan\left(\frac{7\pi}{6}\right)$$

$$50. \cos^{-1}\left(\sin\left(-\frac{\pi}{4}\right)\right)$$

Given the graph of  $y = \sin x$ , sketch the graphs of:

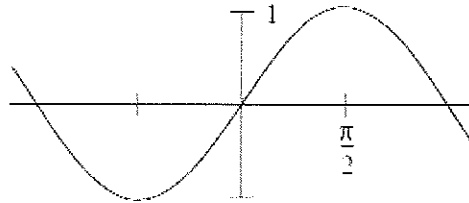
51.  $\sin\left(x - \frac{\pi}{4}\right)$

52.  $\sin\left(\frac{\pi}{2}\right)$

53.  $2 \sin x$

54.  $\cos x$

55.  $\frac{1}{\sin x}$



Solve the equations

56.  $4x^2 + 12x + 3 = 0$

58.  $\frac{x+1}{x} - \frac{x}{x+1} = 0$

57.  $2x + 1 = \frac{5}{x+2}$

Find the remainders on division of

59.  $x^5 - 4x^4 + x^3 - 7x + 1$  by  $x + 2$

60.  $x^5 - x^4 + x^3 + 2x^2 - x + 4$  by  $x^3 + 1$

61. The equation  $12x^3 - 23x^2 - 3x + 2 = 0$  has a solution  $x = 2$ . Find all other solutions.

62. Solve for  $x$ , the equation  $12x^3 + 8x^2 - x - 1 = 0$  (all solutions are rational and between  $\pm 1$ )

Solve the inequalities. Give the solution in interval notation

63.  $x^2 + 2x - 3 \leq 0$

64.  $\frac{2x-1}{3x-2} \leq 1$

65.  $\frac{2}{2x+3} > \frac{2}{x-5}$

Solve for  $x$ . Give the solution in interval notation

66.  $|-x + 4| \leq 1$

67.  $|5x - 2| = 8$

68.  $|2x + 1| > 3$

Determine the equation of the following lines

69. The line through  $(-1, 3)$  and  $(2, -4)$

70. The line through  $(-1, 2)$  and perpendicular to the line  $2x - 3y + 5 = 0$

71. The line through  $(2, 3)$  and the midpoint of the line segment from  $(-1, 4)$  to  $(3, 2)$

72. Find the point of intersection of the lines:  $3x - y - 7 = 0$  and  $x + 5y + 3 = 0$

73. Shade the region in the  $xy$ -plane that is described by the inequalities  $\begin{cases} 3x - y - 7 < 0 \\ x + 5y + 3 \geq 0 \end{cases}$

Find the equations of the following circles:

74. The circle with center at  $(1, 2)$  that passes through the point  $(-2, -1)$

75. The circle that passes through the origin and has intercepts equal to 1 and 2 on the  $x$  and  $y$  axes respectively.

76. For the circle  $x^2 + y^2 + 6x - 4y + 3 = 0$  find the center and the radius

77. Find the domain of  $\frac{3x+1}{\sqrt{x^2+x-2}}$

Find the domain and range of:

78.  $f(x) = 7$

79.  $g(x) = \frac{5x-3}{2x+1}$

80.  $f(x) = \frac{|x|}{x}$

Simplify  $\frac{f(x+h)-f(x)}{h}$  when

81.  $f(x) = 2x + 3$

82.  $f(x) = \frac{1}{x+1}$

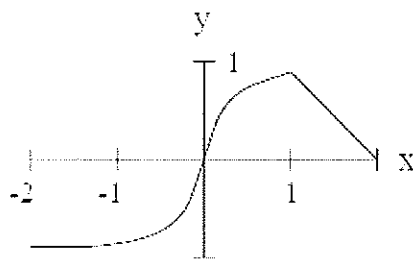
83.  $f(x) = 3x^2 - x + 5$

The graph of the functions  $y = f(x)$  is given as follows: Determine the graphs of the functions:

84.  $f(x + 1)$

85.  $f(-x)$

86.  $|f(x)|$



Sketch the graphs of the functions

87.  $g(x) = |3x + 2|$

88.  $h(x) = |x(x - 1)|$

89. The graph of a quadratic function has x-intercepts  $-1$  and  $3$  and a range consisting of all numbers less than or equal to  $4$ . Determine an expression for the function.

90. Sketch the graph of the quadratic function  $y = 2x^2 - 4x + 3$

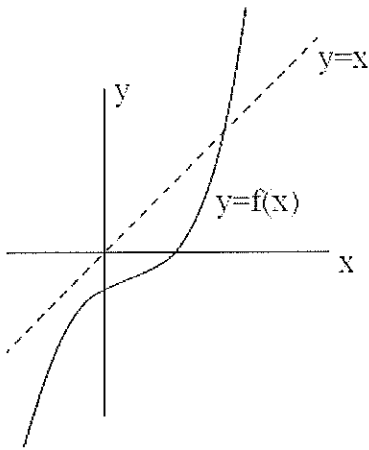
Find the inverse of the functions

91.  $f(x) = 2x + 3$

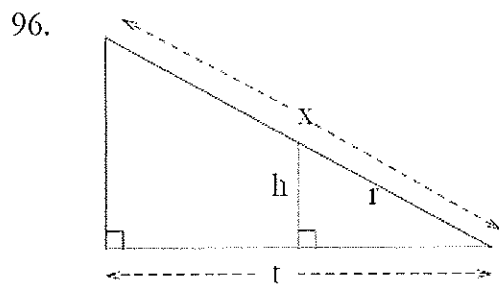
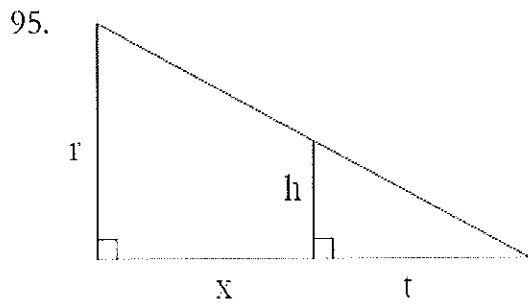
93.  $f(x) = x^2 - 2x - 1, x > 0$

92.  $f(x) = \frac{x+2}{5x-1}$

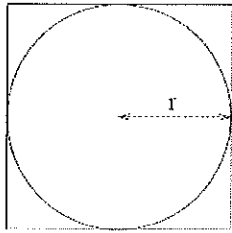
94. A function  $f(x)$  has the graph below. Sketch the graph of the inverse function  $f^{-1}(x)$ .



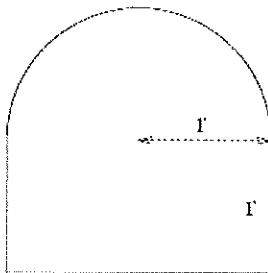
For problems 96 and 97, express  $x$  in terms of the other variables in the picture:



97. Find the ratio of the area inside the square but outside the circle to the area of the square in the picture below



98. Find the formula for the perimeter of the window of the shape in the picture below



99. A water tank has the shape of a cone (like an ice cream cone without the ice cream). The tank is  $10m$  high and has a radius of  $3m$  as the top. If the water is  $5m$  deep (in the middle) what is the surface area of the top of the water?
100. Two cars start moving from the same point. One travels south at  $100\text{ km/hr}$ , the other west at  $50\text{ km/hr}$ . How far apart are they two hours later?
101. A kite is  $100m$  above the ground. If there are  $200m$  of string out, what is the angle between the sting and the horizontal. (Assume that the string is perfectly straight.)

If  $f(x) = 2x - 3$  and  $g(x) = \sqrt{3x - 1}$ , Find:

102.  $f(g(x))$

103.  $g(f(x))$

104. If  $f(x) = \frac{3}{x}$  and  $g(x) = \frac{x}{2x-1}$ , Find  $f(g(x))$  and state its domain.

Decompose each composition function into individual function. (If  $y = f(u)$ , identify  $u$  and rewrite  $y$  in terms of  $u$ )

105.  $y = \sin 3x$

107.  $y = (x^2 - 2x + 5)^5$

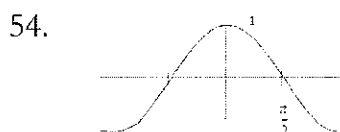
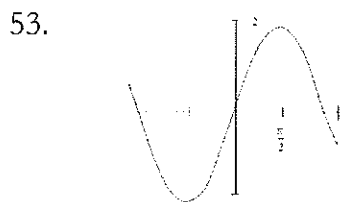
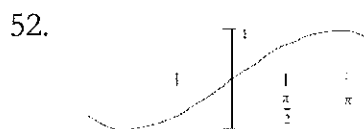
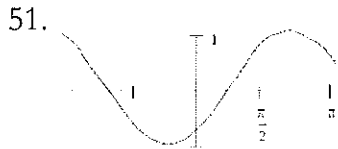
106.  $y = \sqrt[5]{2x + 1}$

108.  $y = \cos^2 x$

## Answers

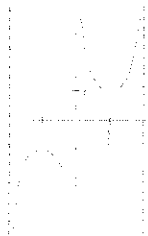
1.  $\frac{x^2+3x}{x-4}$
2.  $\frac{x^2-x}{5x}$
3.  $\frac{x+5}{3x+1}$
4.  $\frac{x}{x}$
5.  $2(\sqrt{3} - \sqrt{2})$
6.  $-1 - \sqrt{5}$
7.  $\frac{1-\sqrt{3}-\sqrt{5}+\sqrt{15}}{-2}$
8.  $8a^6b^{-1}$
9.  $3a^{1/2}b^{3/2}$
10.  $\frac{2}{3}a^2b^{-1}$
11.  $ab^{-1}$
12.  $a^{-3/2}b$
13.  $a^{5/6}b^{1/2}$
14. 1
15.  $-\frac{3}{2}$
16. 8
17.  $\pm \frac{4}{25}$
18.  $\log_2(5(x+1))$
19.  $\log_2 3$
20. 25
21.  $\frac{1}{2}$
22.  $-x$
23.  $2 \log_{10} x$
24.  $\frac{bcx}{bc-cy-bz}$
25.  $\frac{V-2bc}{2(b+c)}$
26.  $\frac{A-2\pi r^2}{2\pi r}$
27.  $\frac{1+\pi r}{2x-y}$
28.  $\frac{x+2y}{\pi}$
29.  $\frac{\pi}{\pi-1}$
30.  $y+1 = (x+2)^2$
31.  $y - \frac{3}{8} = -\frac{3}{2}\left(x + \frac{1}{2}\right)^2$
32.  $x+10 = 9\left(y - \frac{1}{3}\right)^2$
33.  $x^4(x-4)(x+4)$
34.  $(x-2)(2x-5)(2x+5)$

35.  $(2x+3)(4x^2-6x+9)$
36.  $(x-1)(x+1)(x^2+1)$
37.  $0, \pm 4$
38.  $2, \pm \frac{5}{2}$
39.  $-\frac{3}{2}$
40.  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
41.  $-\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$
42.  $\frac{\pi}{6} + 2k\pi$  and  $\frac{5\pi}{6} + 2k\pi$  where  $k \in I$
43.  $-\frac{\sqrt{3}}{2}$
44.  $-\frac{\sqrt{2}}{2}$
45.  $-\frac{\pi}{2}$
46.  $-\frac{4\pi}{2}$
47.  $\frac{\sqrt{2}}{2}$
48.  $\frac{\pi}{3}$
49.  $\frac{\sqrt{3}}{3}$
50.  $\frac{3\pi}{4}$





55.



56.  $\frac{-3 \pm \sqrt{6}}{2}$

57.  $\frac{1}{2}$  or  $-3$

58.  $-\frac{1}{2}$

59.  $-89$

60.  $x^2 + 3$

61.  $-\frac{1}{3}$  or  $\frac{1}{4}$

62.  $-\frac{1}{2}, -\frac{1}{3}, -\frac{1}{2}$

63.  $[-3, 1]$

64.  $(-\infty, \frac{2}{3}) \cup [1, \infty)$

65.  $(-\infty, -8) \cup (-\frac{3}{2}, 5)$

66.  $[3, 5]$

67.  $2$  and  $-\frac{6}{5}$

68.  $(-\infty, -2) \cup (1, \infty)$

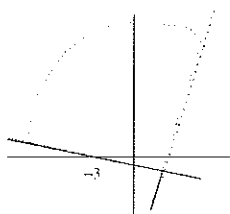
69.  $7x + 3y = 2$

70.  $3x + 2y = 1$

71.  $y = 3$

72.  $(2, -1)$

73.



74.  $(x - 1)^2 + (y - 2)^2 = 18$

75.  $(x - \frac{1}{2})^2 + (y - 1)^2 = \frac{5}{4}$

76. Center =  $(-3, 2)$ , radius =  $\sqrt{10}$

77.  $(-\infty, -2) \cup (1, \infty)$

78. Domain  $(-\infty, \infty)$  Range  $\{7\}$

79. Domain  $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$

Range  $(-\infty, \frac{5}{2}) \cup (\frac{5}{2}, \infty)$

80. Domain  $(-\infty, 0) \cup (0, \infty)$

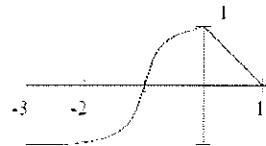
Range  $\{-1, 1\}$

81.  $2$

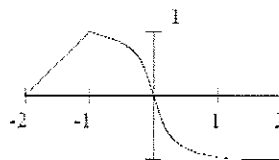
82.  $\frac{-1}{(x+1)(x+h+1)}$

83.  $6x + 3h - 1$

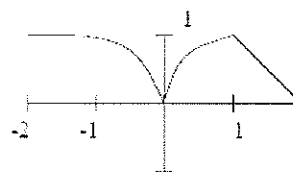
84.



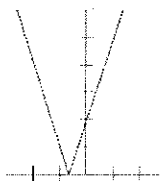
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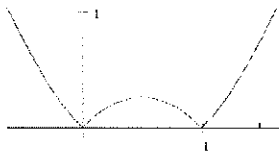
86.



87.

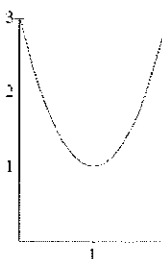


88.



89.  $y = -x^2 + 2x + 3$

90.

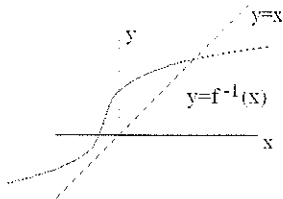


91.  $f^{-1} = \frac{x-3}{2}$

92.  $f^{-1} = \frac{x+2}{5x-1}$

93.  $f^{-1} = 1 + \sqrt{x + 2}$  for  $x > -1$

94.



95.  $x = t \left( \frac{r-h}{h} \right)$

96.  $x = \frac{rt}{\sqrt{r^2 - h^2}}$

97.  $1 - \frac{\pi}{4}$

98.  $4r + \pi r$

99.  $\frac{9\pi}{4}$

100.  $100\sqrt{5} \text{ KM}$

101.  $\frac{\pi}{6}$

102.  $2\sqrt{3x - 1} - 3$

103.  $\sqrt{6x - 10}$

104.  $\frac{6x-3}{x}$

Domain  $(-\infty, 0) \cup \left(0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

105. Let  $u = 3x$ , then  $y = \sin u$

106. Let  $u = 2x + 1$ , then  $y = \sqrt[5]{u}$

107. Let  $u = x^2 - 2x + 5$ ,

then  $y = u^5$

108. Let  $u = \cos x$ , then  $y = u^2$