BERMAN HEBREW ACADEMY

MANDATORY AP CALCULUS SUMMER PACKET

DUE THE FIRST DAY OF SCHOOL

This packet is divided into two parts. **Part I is optional, yet encouraged, and is designed to help you review the necessary Algebra topics for success in AP Calculus.** Many students find the concepts of Calculus interesting and exciting, but do not get the right answers due to Algebra mistakes. Answers to Part I are provided on pp. 7-9. Use your discretion in completing this packet, but please make sure you are comfortable with the material.

Part II (pp. 10-20), a review of limits, is mandatory for all students entering AP Calculus. Students entering AP Calculus from Precalculus+ may want to work with a tutor or take advantage of on-line videos to help them learn the material. We will have a test on this material (plus related new material) early in the year. It is important that you understand the material, not just complete the packet!

Khan Academy is one good on-line resource:

https://www.khanacademy.org/math/differential-calculus/limits_topic

DO ALL PROBLEMS WITHOUT CALCULATORS.

Show all work. Number all problems, if you use a separate sheet of paper. **Part II of packet will be collected and graded.**

Name:_____ Date completed:_____

PART I AP Calculus AB Summer Review Packet

Simplify

1. $\frac{x^3-9x}{x^2-7x+12}$
2. $\frac{x^2-2x-8}{x^3+x^2-2x}$
3. $\frac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$
4. $\frac{9-x^{-2}}{3-x^{-1}}$

Rationalize the denominator

5. $\frac{2}{\sqrt{3}+\sqrt{2}}$
6. $\frac{4}{1-\sqrt{5}}$
7. $\frac{1-\sqrt{5}}{1+\sqrt{3}}$

Write each of the following expressions in the form of ca^pb^q where c, p, and q are numbers

8. $\frac{(2a^2)^3}{b}$
9. $\sqrt{9ab^3}$
10. $\frac{a(2/b)}{3/a}$
11. $\frac{ab-a}{b^2-b}$
12. $\frac{a^{-1}}{(b^{-1})\sqrt{a}}$
13. $\left(\frac{a^{2/3}}{b^{1/2}}\right)^2\left(\frac{b^{3/2}}{a^{1/2}}\right)$

Solve for x. Do not use a calculator

14. $5^{(x+1)} = 25$
15. $\frac{1}{3} = 3^{2x+2}$
16. $\log_2 x = 3$
17. $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$

Simplify

18. $\log_2 5 + \log_2(x^2-1) - \log_2(x-1)$
19. $2\log_4 9 - \log_2 3$
20. $3^{2\log_3 5}$

Simplify

21. $\log_{10} 10^{1/2}$
22. $\log_{10} \frac{1}{10^x}$
23. $2\log_{10}\sqrt{x} + 3\log_{10} x^{1/3}$

Solve the following equations for the indicated variable

24. $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, for a

25. $V=2(ab+bc+ca)$, for a

26. $A=2\pi r^2+2\pi rh$, for positive h

27. $A=P+\pi rP$, for P

28. $2x-2yd=y+xd$, for d

29. $\frac{2x}{4\pi}+\frac{1-x}{2}=0$, for x

For each equation complete the square and reduce to one of the standard forms $y-y_1=A(x-x_1)^2$ or $x-x_1=(y-y_1)^2$

30. $y=x^2+4x+3$

31. $3x^2+3x+2y=0$

32. $9y^2-6y-9-x=0$

Factor completely

33. x^6-16x^4

34. $4x^3-8x^2-25x+50$

35. $8x^3+27$

36. x^4-1

Find all real solutions

37. $x^6-16x^4=0$

38. $4x^3-8x^2-25x+50=0$

39. $8x^3+27=0$

Solve for x

40. $3sin^2x=cos^2x;\quad 0\le x<2\pi$

41. $cos^2x-sin^2x=\sin x;-\pi<x\le\pi$

42. $\tan x+\sec x=2\cos x;-\infty<x<\infty$

Without using a calculator, evaluate the following:

43. $\cos 210°$

44. $\sin\frac{5\pi}{4}$

45. $\tan^{-1}(-1)$

46. $\sin^{-1}(-1)$

47. $\cos\frac{9\pi}{4}$

48. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

49. $\tan\left(\frac{7\pi}{6}\right)$

50. $\cos^{-1}\left(\sin\left(-\frac{\pi}{4}\right)\right)$

Given the graph of $y = \sin x$, sketch the graphs of:

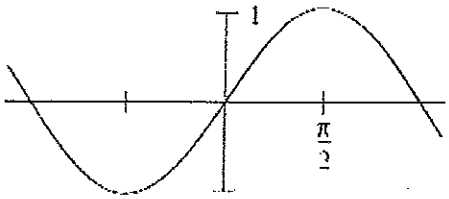
51. $\sin\left(x - \frac{\pi}{4}\right)$

52. $\sin\left(\frac{\pi}{2}\right)$

53. $2\sin x$

54. $\cos x$

55. $\frac{1}{\sin x}$



Solve the equations

56. $4x^2 + 12x + 3 = 0$

57. $2x + 1 = \frac{5}{x+2}$

58. $\frac{x+1}{x} - \frac{x}{x+1} = 0$

Find the remainders on division of

59. $x^5 - 4x^4 + x^3 - 7x + 1$ by $x + 2$

60. $x^5 - x^4 + x^3 + 2x^2 - x + 4$ by $x^3 + 1$

61. The equation $12x^3 - 23x^2 - 3x + 2 = 0$ has a solution $x = 2$. Find all other solutions.

62. Solve for x, the equation $12x^3 + 8x^2 - x - 1 = 0$ (all solutions are rational and between ± 1)

Solve the inequalities. Give the solution in interval notation

63. $x^2 + 2x - 3 \leq 0$

64. $\frac{2x-1}{3x-2} \leq 1$

65. $\frac{2}{2x+3} > \frac{2}{x-5}$

Solve for x. Give the solution in interval notation

66. $|-x + 4| \leq 1$

67. $|5x - 2| = 8$

68. $|2x + 1| > 3$

Determine the equation of the following lines

69. The line through $(-1, 3)$ and $(2, -4)$

70. The line through $(-1, 2)$ and perpendicular to the line $2x - 3y + 5 = 0$

71. The line through $(2, 3)$ and the midpoint of the line segment from $(-1, 4)$ to $(3, 2)$

72. Find the point of intersection of the lines: $3x - y - 7 = 0$ and $x + 5y + 3 = 0$

73. Shade the region in the xy-plane that is described by the inequalities $\begin{cases} 3x - y - 7 < 0 \\ x + 5y + 3 \geq 0 \end{cases}$

Find the equations of the following circles:

74. The circle with center at $(1, 2)$ that passes through the point $(-2, -1)$

75. The circle that passes through the origin and has intercepts equal to 1 and 2 on the x and y axes respectively.

76. For the circle $x^2 + y^2 + 6x - 4y + 3 = 0$ find the center and the radius

77. Find the domain of $\frac{3x+1}{\sqrt{x^2+x-2}}$

Find the domain and range of:

78. $f(x) = 7$

79. $g(x) = \frac{5x-3}{2x+1}$

80. $f(x) = \frac{|x|}{x}$

Simplify $\frac{f(x+h)-f(x)}{h}$ when

81. $f(x) = 2x + 3$

82. $f(x) = \frac{1}{x+1}$

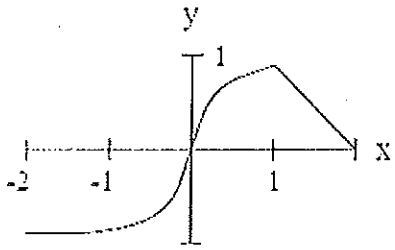
83. $f(x) = 3x^2 - x + 5$

The graph of the functions $y = f(x)$ is given as follows: Determine the graphs of the functions:

84. $f(x + 1)$

85. $f(-x)$

86. $|f(x)|$



Sketch the graphs of the functions

87. $g(x) = |3x + 2|$

88. $h(x) = |x(x - 1)|$

89. The graph of a quadratic function has x-intercepts -1 and 3 and a range consisting of all numbers less than or equal to 4. Determine an expression for the function.

90. Sketch the graph of the quadratic function $y = 2x^2 - 4x + 3$

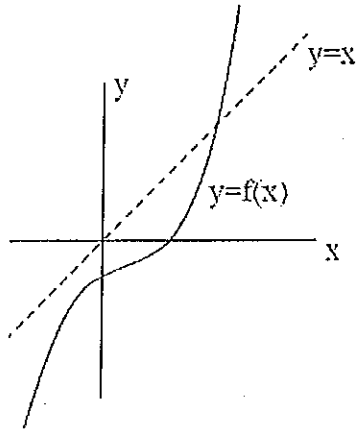
Find the inverse of the functions

91. $f(x) = 2x + 3$

92. $f(x) = \frac{x+2}{5x-1}$

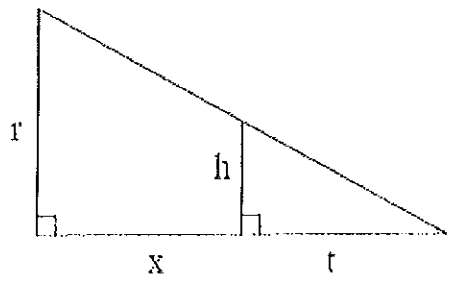
93. $f(x) = x^2 - 2x - 1, x > 0$

94. A function $f(x)$ has the graph below. Sketch the graph of the inverse function $f^{-1}(x)$.

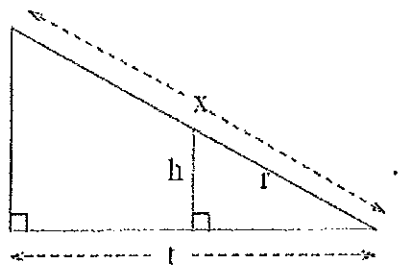


For problems 96 and 97, express x in terms of the other variables in the picture:

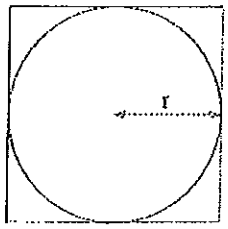
95.



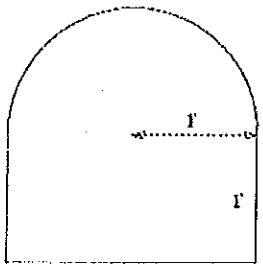
96.



97. Find the ration of the area inside the square but outside the circle to the area of the square in the picture below



98. Find the formula for the perimeter of the window of the shape in the picture below



99. A water tank has the shape of a cone (like an ice cream cone without the ice cream). The tank is $10m$ high and has a radius of $3\ m$ as the top. If the water is $5\ m$ deep (in the middle) what is the surface area of the top of the water?

100. Two cars start moving from the same point. One travels south at $100\ km/hr$, the other west at $50\ km/hr$. How far apart are they two hours later?

101. A kite is $100\ m$ above the ground. If there are $200\ m$ of string out, what is the angle between the sting and the horizontal. (Assume that the string is perfectly straight.)

If $f(x) = 2x - 3$ and $g(x) = \sqrt{3x - 1}$, Find:

102. $f(g(x))$

103. $g(f(x))$

104. If $f(x) = \frac{3}{x}$ and $g(x) = \frac{x}{2x-1}$, Find $f(g(x))$ and state its domain.

Decompose each composition function into individual function. (If $y = f(u)$, identify $u$and rewrite y in terms of u)

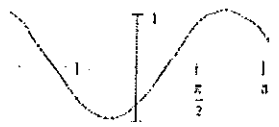
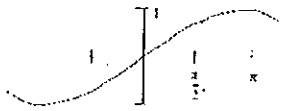
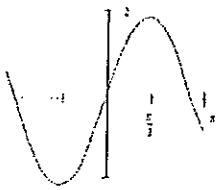
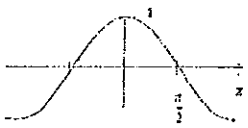
105. $y = \sin 3x$

106. $y = \sqrt[5]{2x + 1}$

107. $y = (x^2 - 2x + 5)^5$

108. $y = cos^2 x$

Answers

1. $\frac{x^2+3x}{x-4}$
2. $\frac{x-4}{x^2-x}$
3. $\frac{5x}{x+5}$
4. $\frac{3x+1}{x}$
5. $2(\sqrt{3}-\sqrt{2})$
6. $-1-\sqrt{5}$
7. $\frac{1-\sqrt{3}-\sqrt{5}+\sqrt{15}}{-2}$
8. $8a^6b^{-1}$
9. $3a^{1/2}b^{3/2}$
10. $\frac{2}{3}a^2b^{-1}$
11. ab^{-1}
12. $a^{-3/2}b$
13. $a^{5/6}b^{1/2}$
14. 1
15. $-\frac{3}{2}$
16. 8
17. $\pm\frac{4}{25}$
18. $\log_2(5(x+1))$
19. $\log_2 3$
20. 25
21. $\frac{1}{2}$
22. $-x$
23. $2\log_{10} x$
24. $\frac{bcx}{bc-cy-bz}$
25. $\frac{V-2bc}{2(b+c)}$
26. $\frac{A-2\pi r^2}{2\pi r}$
27. $\frac{A}{1+\pi r}$
28. $\frac{2x-y}{x+2y}$
29. $\frac{\pi}{\pi-1}$
30. $y+1=(x+2)^2$
31. $y-\frac{3}{8}=-\frac{3}{2}\left(x+\frac{1}{2}\right)^2$
32. $x+10=9\left(y-\frac{1}{3}\right)^2$
33. $x^4(x-4)(x+4)$
34. $(x-2)(2x-5)(2x+5)$
35. $(2x+3)(4x^2-6x+9)$
36. $(x-1)(x+1)(x^2+1)$
37. $0, \pm 4$
38. $2, \pm\frac{5}{2}$
39. $-\frac{3}{2}$
40. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
41. $-\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$
42. $\frac{\pi}{6}+2k\pi$ and $\frac{5\pi}{6}+2k\pi$ where $k \in I$
43. $-\frac{\sqrt{3}}{2}$
44. $-\frac{\sqrt{2}}{2}$
45. $-\frac{\pi}{4}$
46. $-\frac{\pi}{2}$
47. $\frac{\sqrt{2}}{2}$
48. $\frac{\pi}{3}$
49. $\frac{\sqrt{3}}{3}$
50. $\frac{3\pi}{4}$
51. 
52. 
53. 
54. 

55.

56. $\frac{-3\pm\sqrt{6}}{2}$

57. $\frac{1}{2}$ or -3

58. $-\frac{1}{2}$

59. -89

60. x^2+3

61. $-\frac{1}{3}$ or $\frac{1}{4}$

62. $-\frac{1}{2},-\frac{1}{3},-\frac{1}{2}$

63. $[-3,1]$

64. $\left(-\infty,\frac{2}{3}\right)\cup[1,\infty)$

65. $(-\infty,-8)\cup\left(-\frac{3}{2},5\right)$

66. $[3,5]$

67. 2 and $-\frac{6}{5}$

68. $(-\infty,-2)\cup(1,\infty)$

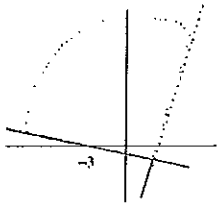
69. $7x+3y=2$

70. $3x+2y=1$

71. $y=3$

72. $(2,-1)$

73.



74. $(x-1)^2+(y-2)^2=18$

75. $\left(x-\frac{1}{2}\right)^2+(y-1)^2=\frac{5}{4}$

76. Center $=(-3,2)$, radius $=\sqrt{10}$

77. $(-\infty,-2)\cup(1,\infty)$

78. Domain $(-\infty,\infty)$ Range $\{7\}$

79. Domain $\left(-\infty,-\frac{1}{2}\right)\cup\left(-\frac{1}{2},\infty\right)$
Range $\left(-\infty,\frac{5}{2}\right)\cup\left(\frac{5}{2},\infty\right)$

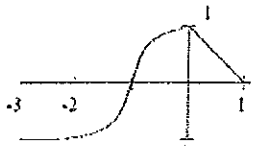
80. Domain $(-\infty,0)\cup(0,\infty)$
Range $\{-1,1\}$

81. 2

82. $\frac{-1}{(x+1)(x+h+1)}$

83. $6x+3h-1$

84.



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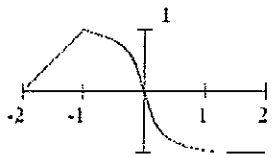

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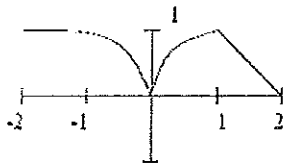

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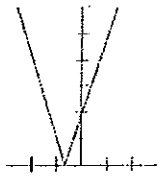

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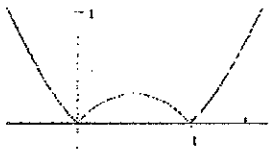
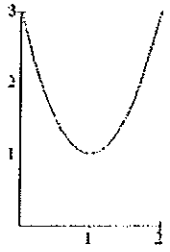


89. $y=-x^2+2x+3$

90.

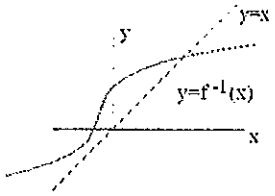


91. $f^{-1}=\frac{x-3}{2}$

92. $f^{-1}=\frac{x+2}{5x-1}$

93. $f^{-1} = 1 + \sqrt{x+2}$ for $x > -1$

94.



95. $x = t\left(\frac{r-h}{h}\right)$

96. $x = \frac{rt}{\sqrt{r^2-h^2}}$

97. $1 - \frac{\pi}{4}$

98. $4r + \pi r$

99. $\frac{9\pi}{4}$

100. $100\sqrt{5}\ KM$

101. $\frac{\pi}{6}$

102. $2\sqrt{3x-1} - 3$

103. $\sqrt{6x-10}$

104. $\frac{6x-3}{x}$

Domain $(-\infty, 0) \cup \left(0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

105. Let $u = 3x$, then $y = \sin u$

106. Let $u = 2x + 1$, then $y = \sqrt[5]{u}$

107. Let $u = x^2 - 2x + 5$,
then $y = u^5$

108. Let $u = \cos x$, then $y = u^2$

PART II:

Limit as x approaches infinity

1. $\lim_{x\to\infty}\left(\frac{3x-7}{5x^4-8x+12}\right)=$

2. $\lim_{x\to\infty}\left(\frac{3x^4-2}{5x^4-2x+1}\right)=$

3. $\lim_{x\to\infty}\left(\frac{x^6-2}{10x^4-9x+8}\right)=$

4. $\lim_{x\to\infty}\left(\frac{7x^4-2}{5-2x^3-14x^4}\right)=$

5. $\lim_{x\to\infty}\left(\frac{\sin x}{e^x}\right)=$

6. $\lim_{x\to-\infty}\left(\frac{\sqrt{x^2-9}}{2x-3}\right)=$

7. $\lim_{x\to\infty}\left(\frac{\sqrt{x^2-9}}{2x-3}\right)=$

Limit as x approaches a number

8. $\lim_{x \to 2} \left(x^3 - x + 1\right)$

9. $\lim_{x \to 2} \left(\frac{x^2 - 4}{x - 2}\right) =$

10. $\lim_{x \to 2^+} \left(\frac{3}{x - 2}\right) =$

11. $\lim_{x \to 2^-} \left(\frac{3}{x - 2}\right) =$

12. $\lim_{x \to 2} \left(\frac{3}{x - 2}\right) =$

13. $\lim_{x \to 2^-} \left(\frac{3}{2 - x}\right) =$

14. $\lim_{x \to \frac{\pi}{4}} \left(\frac{\sin x}{x}\right) =$

15. $\lim_{x \to \frac{\pi}{4}} \left(\frac{\tan x}{x}\right) =$

recall: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

1. What is $\lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h}$?

(A) $\sin x$ (B) $\cos x$ (C) $-\sin x$

(D) $-\cos x$ (E) The limit does not exist

2. $\lim_{\Delta x \to 0} \frac{\cos\left(\frac{\pi}{3} + \Delta x\right) - \cos\left(\frac{\pi}{3}\right)}{\Delta x} =$

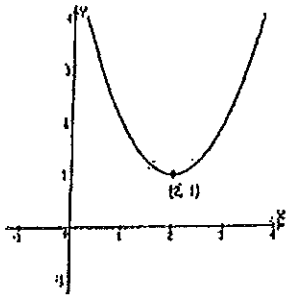
(A) $-\frac{\sqrt{3}}{2}$ (B) $-\frac{1}{2}$ (C) 0

(D) $\frac{1}{2}$ (E) $\frac{\sqrt{3}}{2}$

3. $\lim_{h \to 0} \frac{(x+h)^3 - (x^3)}{h} =$

(A) $-x^3$ (B) $-3x^2$ (C) $3x^2$

(D) x^3 (E) The limit does not exist



4. The graph of $y = f(x)$ is shown above. $\lim_{x \to 2} \left(\left(f(x)^3 \right) - 3f(x) + 7 \right)$

(A) 1 (B) 5 (C) 7 (D) 9 (E) Do

5. If $f(x) = \begin{cases} \dfrac{x^2 - 3x - 4}{x+1}, & x \neq -1 \\ 2, & x = -1 \end{cases}$, what is $\lim_{x \to -1} f(x)$?

(A) -5 (B) 0 (C) 2 (D) 3 (E) Do

6. $\lim_{x \to \infty} \left(\dfrac{2x^6 - 5x^3 + 10}{20 - 4x^2 - x^6} \right) =$

(A) -2 (B) $-\dfrac{1}{2}$ (C) $\dfrac{1}{2}$ (D) 2 (E) Do

7. $\lim_{x \to \infty} \left(\dfrac{2x^5 - 5x^3 + 10}{20 - 4x^2 - x^6} \right) =$

(A) -2 (B) $-\dfrac{1}{2}$ (C) 0 (D) $\dfrac{1}{2}$ (E) 2

8. $\lim_{x\to\infty}\left(1+e^{\frac{1}{x^2}+\frac{1}{x}}\right)=$

(A) $-\infty$ (B) 0 (C) $e^{\frac{1}{2}}$

(D) $1+e^{\frac{1}{2}}$ (E) ∞

9. $\lim_{x\to 3^+}\frac{5}{3-x}=$

(A) $-\infty$ (B) -5 (C) 0

(D) $\frac{5}{3}$ (E) ∞

10. If $\lim_{x\to\infty}\left(\frac{5n^3}{20-3n-kn^3}\right)=\frac{1}{2}$, then $k=$

(A) -10 (B) -4 (C) $\frac{1}{4}$ (D) 4 (E) 10

11. Which of the following is/are true about the function g if $g(x)=\frac{(x-2)^2}{x^2+x-6}$

I. g is continuous at $x=2$

II. The graph of g has a vertical asymptote at $x=-3$

III. The graph of g has a horizontal asymptote at $y=0$

(A) I only (B) II only (C) III only (D) I and II only

12. $f(x)=\begin{cases}\sin x, & x<\frac{\pi}{4}\\ \cos x, & x>\frac{\pi}{4}\\ \tan x, & x=\frac{\pi}{4}\end{cases}$

What is $\lim_{x\to\frac{\pi}{4}} f(x)$?

(A) $-\infty$ (B) 0 (C) 1 (D) $\frac{\sqrt{2}}{2}$ (E) ∞

13. $\lim_{x\to a}\left(\frac{\sqrt{x}-\sqrt{a}}{x-a}\right)=$

(A) $\frac{1}{2\sqrt{a}}$ (B) $\frac{1}{\sqrt{a}}$ (C) $\sqrt{a}$ (D) $2\sqrt{a}$ (E) DNE

14. $\lim_{x\to 0^+}\frac{\ln 2x}{2x}=$

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

15. At $x=4$, the function given by $h(x)=\begin{cases}x^2, & x\le 4\\ 4x, & x>4\end{cases}$ is

(A) Undefined
(B) Continuous but not differentiable
(C) Differentiable but not continuous
(D) Neither continuous nor differentiable
(E) Both continuous and differentiable

Free Response 2 (No calculator)

Given the function $f(x)=\dfrac{x^3+2x^2-3x}{3x^2+3x-6}$.

(a) What are the zeros of $f(x)$?

(b) What are the vertical asymptotes of $f(x)$?

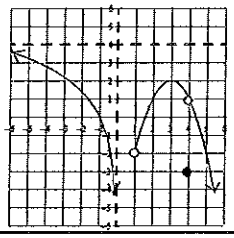
(c) The end behavior model of $f(x)$ is the function $g(x)$. What is $g(x)$?

(d) What is $\lim_{x\to\infty} f(x)$? What is $\lim_{x\to\infty} \dfrac{f(x)}{g(x)}$?

For question 12 – 16, use the equation $g(x)$ below and the graph of the function $f(x)$.

$$g(x)=\begin{cases}3|x+3|, & x<-2\\ \cos\left(\frac{\pi x}{2}\right), & -2\le x<2\\ ax^2+2x, & x\ge 2\end{cases}$$

Graph of $f(x)$



12. Is $g(x)$ continuous at $x=-2$. [Base your response on the three part definition of continuity.]

13. For what value(s) of a is $g(x)$ continuous at $x=2$?

14. For what value(s) of b is the function $f(x)$ discontinuous? At which of these values does $\lim_{x\to b} f(x)$ exist? Explain your reasoning.

15. Find $\lim_{x\to 3}[g(x)+2f(x)]$.

16. Which of the following limits do(es) not exist? Give a reason for your answers.

$\lim_{x\to 1} f(x)$	$\lim_{x\to 4} f(x)$	$\lim_{x\to 0^-} f(x)$